Unit-V

Kinetics and Mechanical Vibrations

1) State D'Alembert's principle?

A) D'Alembert's principle state that a moving body can be brought to equilibrium by adding inertia force to the system. In magnitude, this inertia force is equal to the product of mass and acceleration and takes place in a direction opposite to that of acceleration

(Or)

For any body, the algebraic sum of externally applied forces and forces resisting motion in any direction is zero. $\Sigma F - \Sigma F_i = 0$

$$\Sigma F - ma = 0$$

$-ma = \Sigma F_i$ is inertia force

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2) State the principle of Impulse-momentum

A) The impulse-momentum theorem state that change in momentum (ΔP) of a ~~static~~ object equals the impulse (J) applied to it.

Impulse (J) = change in momentum (ΔP)

$J = \bar{F}\Delta t$ / $\bar{F}$ = average net force acting on object

If mass is constant $\bar{F}\Delta t = m\Delta v$

If mass is changing, then

$$F dt = m dV + V dm$$

The impulse momentum theorem is logically equivalent to Newton's 2nd law of motion

Units: S.I Units of Impulse is Ns.
S.I unit of momentum is $Kg \frac{m}{s}$

Units of impulse momentum are equivalent

$$\left(Ns = Kg \frac{m}{s}\right)$$

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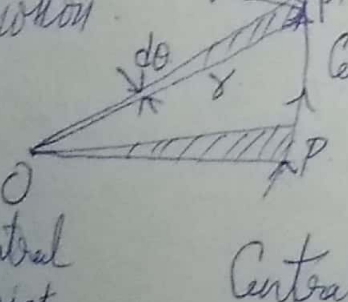
* What do you understand by Central force ~~motion~~ motion?

A) The motion of a particle under the action of a force directed towards (or) away from a fixed point is termed as Centre force motion. The force which is directed towards or away from fixed point is called as central force.

In general, central force motion may take place along trajectory, e.g. ~~Thought~~ Throwing of stone an angle inclined to horizontal direction (or) a periodic motion of satellite.



Central force motion of P.

Central force motion

* The motion of a particle in rectilinear motion is defined by relation $S = 2t^3 - 9t^2 + 12t - 10$ where S is expressed in meters and t in seconds. Find the acceleration of a particle when the velocity is zero

A).

$$S = 2t^3 - 9t^2 + 12t - 10$$

$$\frac{ds}{dt} = V = 2(3t^2) - 9(2t) + 12$$

$$V = 6t^2 - 18t + 12$$

$$\frac{dv}{dt} = 12t - 18 \quad | \quad a = 12t - 18.$$

when $V=0$, $0 = 6t^2 - 18t + 12$

$6t^2 - 12t - 6t + 12 = 0$

$6t^2 - 6t - 12t + 12 \quad | \quad 6t(t-1) - 12(t-1) = 0$

$(6t-12)(t-1) = 0 \quad | \quad t-1=0, t=1.$

$6t - 12 = 0.$

$6t = 12, t = 2$

$\therefore t = 2, 1.$

To find acceleration, when $V=0$

Case ① $a = 12t - 18 = 12(2) - 18 = 6 \; m/s^2$
Put $t=2$

Case ② Put $t=1$, $a = 12(1) - 18 = -6 \; m/s^2$ (acceleration would not be -ve)

So acceleration $= 6 m/s^2$

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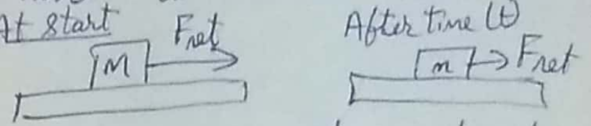
* Derive the impulse-Momentum equation of a body in motion

A). Suppose a constant Force (F_{net}) is applied to an object of mass (m).

Newton 2nd law says that object will accelerate if it starts with velocity V_i after some time (t) its velocity will be V_o . .

The acceleration of object equals its change in velocity divided by the time it takes the velocity to change. $a = \frac{\Delta V}{t}$



Multiplying both sides of this equation by t gives;

$$at = \Delta V = V_o - V_{i}$$

The right side of equation above comes from the fact that the change in velocity equals the final velocity V_o, minus the starting velocity V_i

$$at = V_o - V_i \quad / \quad V_o = V_i + at$$

— ① This is Kinematic equation. about motion

To turn into dynamic statement (equation) about motion multiply both sides of equation by object mass, m

$$mV_i + mat = mV_o$$

$$mat = m(V_o - V_i) = P_o - P_{i}$$

$$F_{net}(t) = P_o - P_{i}$$

$F = ma$ $\quad$ $P = mV$

$(t)(F_{net})$ is impulse exerted on object by net force.

The quantity on right of equation is final momentum minus its starting momentum, which is its change in momentum

Impulse $= \Delta P$.: This is Impulse - Momentum Equation.

* Define Momentum and Impulse

A) Impulse:- Impulse is change of momentum of an object when the object is acted upon by a force for an interval of time

(or)

Impulse is integral of force F, over the time interval t, for which it acts. Since it is force is vector quantity. Impulse is also vector quantity as it is applied to an object produces an equivalent vector change in its linear momentum & also in same direction Units: Ns (or) kg m/s

Momentum:-

Momentum is defined as "mass in motion". "All objects have mass. So if an object is moving, then it has momentum & it has its mass in motion. The amount of momentum that an object is dependent upon two variables; how much stuff is moving and how fast the stuff is moving.

Momentum depends upon variables mass & velocity In terms of an equation, the momentum of an object is equal to the mass of object times the velocity of object.

Momentum = mass x velocity

$P = mv$

* Distinguish b/w Kinematics & Kinetics?

A)

Kinetics	Kinematics
1) It deals with the causes for motions of object	1) It deals with position, acceleration, speed of an object
2) It takes into consideration of the mass of object	2) It doesn't consider the mass of the object
3) Its practical applications can be found in the designing of automobiles	3) Its application can be found in study of movement of celestial bodies (astronomical objects)
4) It takes force explicitly into account (details)	4) It doesn't take force explicitly into account
5). It doesn't have more mathematical expressions	5) It has more mathetical expressions
6) This topic is used in various branches such as biology, chemistry, and physics	6) This topic is used in physics, mechanics ~~and~~ in terms of engineering

* The potential energy of body is 39600J. How high is body if its mass is 20kg?

③

A) Given $P.E = 39600J$.

$P.E = mgh$. Take $g = 10m/s^2$.

$39600 = 20 \times 10 \times h$ / $h = 198\,m$

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* Derive the expression for work-energy of a body in motion?

A). Consider the body of mass 'm' is moving with a velocity v. A force 'F' is applied on it to stop its motion for which the retardation is produce in the body and body covers a distance before coming in rest.

Work done by force to stop the motion of body is measure of kinetic energy.

Here initial velocity v and final velocity = 0.

From Newton's 2nd law

$F = ma$

$a = F/m$.

Using constant acceleration equation.

$\cancel{0^2 - v^2 = 2as}$ $0^2 - v^2 = 2as$.

$0^2 - 2as = v^2$, $a = -\frac{v^2}{2as}$

Force acting on the body

$F = ma = -\frac{mv^2}{2as}$

By Newton's third law of motion, the body would exert an equal force in the opposite direction

$\therefore$ Force exerted by body $= \frac{mv^2}{2as}$

The kinetic energy possessed by body is equal to work done before it comes to rest.

Work done by the body = $F \times S$ ⑧

Kinetic energy = $F \times S = \frac{mv^2}{2S} \times S = \frac{1}{2} mv^2$

$K.E = \frac{1}{2} (mv^2)$

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* What is power? How do you differentiate Kilowatt from Kilowatt hour?

A). Power is rate of doing work.

Kilowatt is unit power and Kilowatt hour is bigger unit of energy

(Kilowatt = 1000W)

1KWh = 3600 × 1000

= 3.6×10^6 J. = 3600 KJ.

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* Differentiate b/w curvilinear motion and Rectilinear motion?

A)

Curvilinear motion	Rectilinear motion
1) When a body moves along a curved path then the linear motion is called curvilinear motion.	1) When a body moves in a straight line then the linear motion is called rectilinear motion.
2) In this, position, velocity acceleration of a particle is considered	2) In this, position, velocity acceleration of a particle is considered
Ex:- Kicked football, Car travelling turning a corner	Ex:- Sprinter running, sliding cell phone across a table

* State work energy equation for translation?

A). When a force does work on a body, the increase in Kinetic energy of body is equal to amount of work done by force (or decrease in Kenetic energy if work done is negative)

Statement of Work - energy theorem (principle)

The Work done by resultant force acting on a body is equal to the change in Kinetic energy of the body

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2}$$

* State the assumptions made while studying projectile motion

A) 1) Acceleration due to gravity 'g' is constant both in magnitude and direction

2) There is no resistance due to air

3) The earth is flat

4) Rotational ~~~~ motion of earth is absent

* State the law of conservation of momentum?

A) It states that For two or more bodies in an isolated system acting upon each-other, their total momentum remains constant unless an external force is applied. Therefore momentum can be neither be created nor destroyed.

Conservation of momentum is derived from Newton's 3rd law of motion. Newton's 3rd law of motion states that every action has an equal but opposite reaction; the force that one object A exerts on object B is equal but opposite to force that object B exerts on object A.

Note ÷ Isolated system is system in which no mass & no energy transfers

* What are the parameters that define rectilinear motion? State the relationship b/w these parameters?

A) Parameters that involved in a rectilinear motion are displacement, velocity, acceleration and time.

$$V = \frac{dx}{dt}$$

$$a = \frac{d^2x}{dt^2}$$

* Derive the equation of work energy

A) Let a Force F acting on a body of mass 'm' moving with an initial velocity u. Let v be final velocity

Let V be final velocity (v) after time 't' Let S be the distance travelled through which body moves during the time 't'

$\boxed{v^2 - u^2 = 2as}$ as a is acceleration is produced

Multiplying the Equation by $\left(\frac{1}{2}\right)m$ we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m \times 2as.$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas.$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = F \times S$$

$F \times S =$ Workdone

$$\boxed{W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = FS}$$

Or.

$$v^2 - u^2 = 2as$$

$$S = \frac{v^2 - u^2}{2a}$$

$F = ma$

$F \times S = ma \times S.$

$$F \times S = ma\left(\frac{v^2 - u^2}{2a}\right)$$

$$\boxed{W.D = \frac{1}{2}mav^2 - \frac{1}{2}mu^2}$$

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* Define Fixed axes of rotation and give an example

A). Fixed axis of rotation describes the rotation around a fixed axis of a rigid body

A rigid body is said to be in fixed axis rotation, if there exists a fixed straight line within (or) outside the body such that the points identified with the body but on that line have zero velocity and zero acceleration.

The Rotation may also be specified to be about a point if there exists only a fixed point identified with the body where both the velocity and accelera-
-tion vanish

An example of fixed-axis rotation is (a) shaft rotating in a fixed journal bearing
(b) Spinning top rotating about tip in steady (or) unsteady states.

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* Define general plane motion and give an example

A) The motion of a rigid body is said to be plane motion if all the points in a body stay in the same parallel planes. A plane motion may be composed of translation and rotation.

Example

1) Linear translation of a rigid body must be plane motion.
2) Rotation of a rigid body about a fixed axis must be plane motion

* Define motion and different types of motion.

A) When a body does not change its position with time, then the body is at rest while, if the body change its position with time, it is said to be in motion

<u>Types of motion</u>

1) translational motion - The object move same distance in a given time
2) Rotational motion - The object moves about an axes
3) Periodic motion - motion that repeats itself after equal intervals of time
4) Non periodic motion - A motion does not repeat itself at regular intervals.

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* The maximum range of projectile is 2000m. What should be angle of elevation so as to obtain a range of 1400m. If the initial velocity remains unchanged?

A) For maximum horizontal range of projectile, angle of elevation is 45°. ($\alpha = 45^\circ$), Given $R = 2000m$

$$R = \frac{U^2 \sin 2(\alpha)}{g}$$

$$\frac{2000 \times 9.81}{\sin 2(45)} = U^2 \quad | \quad U = 140.07 \, m/s$$

Case (2). $R = 1400$, $U = 140.07$ (initial velocity unchanged)

$\frac{R \times g}{U^2} = \sin 2\alpha \quad | \quad \alpha = \frac{\sin^{-1}\left(\frac{R \times g}{U^2}\right)}{2} = 21^\circ.96'$

* Derive work energy equation for translation

A). The general problem in dynamics is to determine the relation b/w force system and motion of body (or) system. For a particle (or) translating rigid body, we have to solve the equation $R = ma$ for the acceleration and then integrate one of the equations

$$a_t = \frac{dv}{dt}, \quad v = \frac{ds}{dt} \quad \text{(or)} \quad a\,ds = dv, \text{ where}$$

R = resultant of force system, using relation

$R = ma$ for motion of translation

$\therefore$ we have $R = ma$

$$R = \frac{m\,dv}{dt} = m\frac{dv}{ds} \times \frac{ds}{dt} = m\frac{dv}{ds} \times v$$

by transposing, we get

$$R\,ds = m v\,dv \quad \text{---(a)}, \quad \text{Integrating eq (a)}$$

$$\int_{s_1}^{s_2} R\,ds = \int_{v_1}^{v_2} m v\,dv = \frac{1}{2} m (v_2^2 - v_1^2)$$

The left hand side is known as resultant work (W_{1-2}) done upon the particle and the term $\frac{1}{2}v^2$ is called kinetic energy.

$$W_{12} = \frac{1}{2} m (v_2^2 - v_1^2) = \Delta KE$$

$$W_{1-2} = \frac{1}{2} m (V_2^2 - V_1^2) = \Delta KE$$

This is known as work-energy equation for a particle. It states that the resultant workdone upon a particle between positions 1 and 2 is equal to change in kinetic energy.

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* State work-energy theorem for a system of particles?

A) **Work-energy theorem for a system of particles :-**

It states that change in kinetic energy of a system of particles is equal to sum of workdone by internal forces and workdone by external forces.

$$\boxed{\Delta K.E_{system} = W_{\substack{internal \\ forces}} + W_{\substack{external \\ forces}}}$$

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* What is importance of Impulse-momentum method?

A) 1) Impulse momentum method directly relates force, velocity and time.

2). It is particularly useful in situations when forces act for a very small time intervals during which forces may vary, as in sudden blow, collision or impact.

3) It is also useful in which the System gains or loss mass.

* Explain work-energy method for a plane motion

A) If a rigid body moves with both translational and rotational motion, then it is said to be in general plane motion.

$$\boxed{KE_1 + \Sigma Works = KE_2}$$

$\Sigma W = W_{\text{internal forces}} + W_{\text{external forces}}$.

This Equation states that body's initial translational and rotation Kinetic energy plus the work done by all external forces and couple moments acting on a body as the body moves on a plane from its initial to its final position is equal to body's final translational and rotational Kinetic energy.

Note: The work of the body's internal forces does not have considered since the body is rigid at starting point.

Thanks for reading, Hope this will help you.

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